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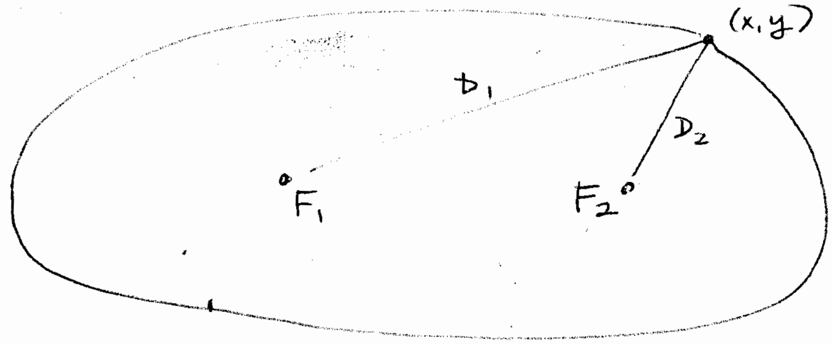
The Ellipse and The Hyperbola

Lessons 50 & 51

1) Recognize equation of ellipse
2) Graph ellipse
3) Recognize equation of hyperbola
4) Graph hyperbola

An ellipse is the set of all points (x, y) such that the sum of of the two distances from (x, y) to point F_1 and from (x, y) to point F_2. is a constant



The points F_1 and F_2 are called <u>foci</u> ("foh-sigh"), the plural of focus.

① Sketch $\frac{x^2}{9} + \frac{y^2}{16} = 1$

Coefficient	coefficient
$\frac{1}{9}$	$\frac{1}{16}$
$\frac{1}{9}x^2$	$\frac{1}{16}y^2$

Notice

1. x^2 and y^2
2. sum
3. coefficients are different
4. equal 1.

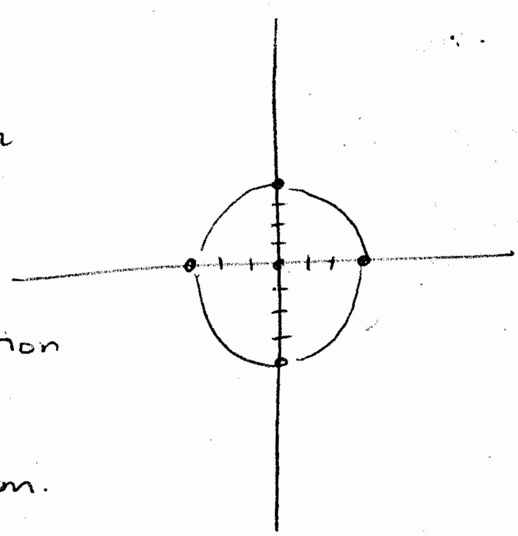
x^2 means $(x-0)^2$
X coord of center is 0.

y^2 means $(y-0)^2$
y coord of center is 0.

<u>Step 1</u> plot center

9 below $x^2 \Rightarrow \sqrt{9} = 3$ units in x-direction
<u>Step 2:</u> plot 2 points left & right

16 below $y^2 \Rightarrow \sqrt{16} = 4$ units in y direction.
<u>step 3:</u> plot 2 points up & down

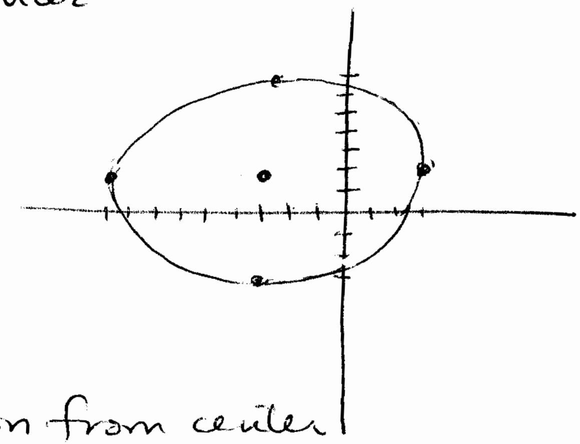


<u>Step 4:</u> Fill in oval shape

② Sketch $\frac{(x+3)^2}{36} + \frac{(y-2)^2}{25} = 1$

$x+3 \Rightarrow x = -3$ x-coord of center
$y-2 \Rightarrow y = 2$ y coord of center

Step 1: plot $(-3, 2)$



36 below $(x+3)^2$
step 2: $\Rightarrow \sqrt{36} = 6$ units left and right from center

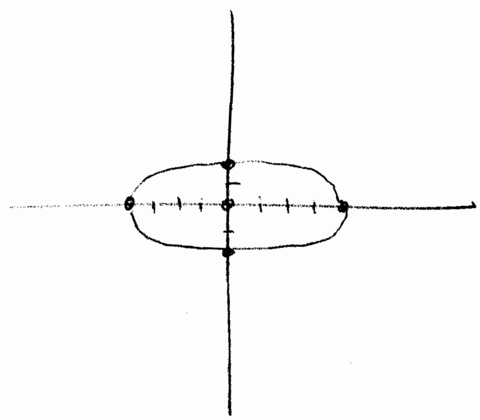
25 below $(y-2)^2$
step 3: $\Rightarrow \sqrt{25} = 5$ units up and down from center
step 4: fill in curves

③ Sketch graph $4x^2 + 16y^2 = 64$

Step 0: Notice that it's not equal to 1.
Focus on correcting the RHS and the other coefficients will fix themselves.

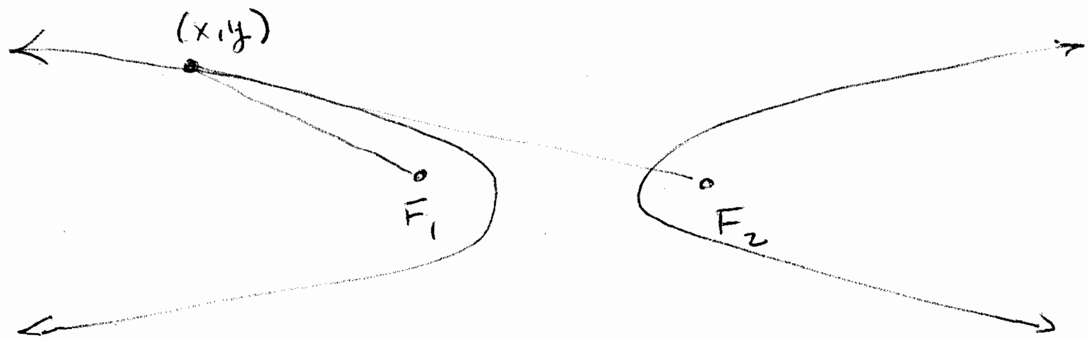
$$\frac{4x^2}{64} + \frac{16y^2}{64} = \frac{64}{64}$$

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$



step 1: center $(0,0)$
step 2: x direction $\sqrt{16} = 4$
step 3: y direction $\sqrt{4} = 2$

An hyperbola is the set of all points (x,y) such that the difference of the two distances from (x,y) to point F_1 and from (x,y) to point F_2 is a constant



The points F_1 and F_2 are also called foci.

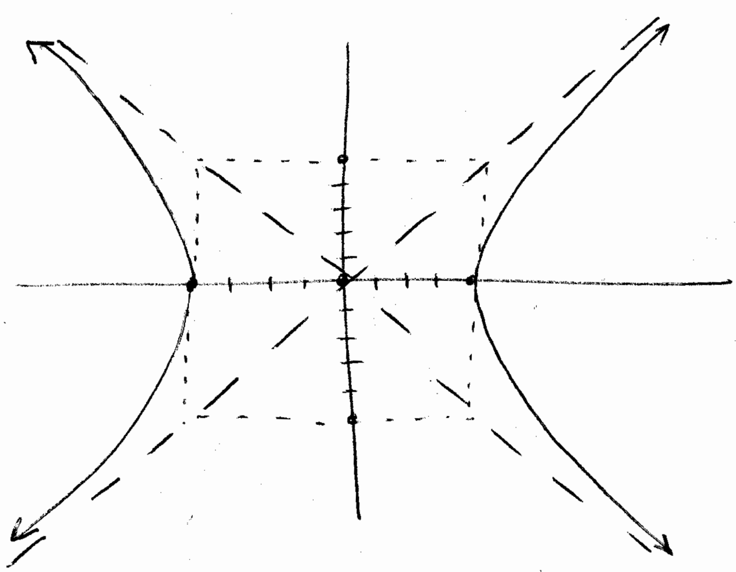
(4) Sketch $\frac{x^2}{16} - \frac{y^2}{25} = 1$

step 1: Find center the same way as ellipse
$(0,0)$

step 2: Plot 4 points in x dir and y dir same as ellipse

Notice
1. x^2 and y^2
2. difference
3. equal to 1
{coefficients are irrelevant}
4. Notice which variable appears first
$x^2 - y^2$ open left/right x first
$y^2 - x^2$ open up/down y first

-- FROM HERE THE PROCESS IS TOTALLY DIFFERENT ----



step 3: Draw a dashed line box through the four points.

Step 4: Connect corners to draw asymptotes

step 5: If $x^2 - y^2$ draw left + right branches through the points on the box, called vertices.

Must have vertices + asymptotes

⑤ Sketch

$$\frac{(y+3)^2}{4} - \frac{(x-1)^2}{9} = 1$$

hyperbola because subtracted

up/down because y^2 appears first

center: numerators

x-coord next to x-variable

take opposite sign

$x = 1$

y-coord next to y-variable

take opposite sign

$y = -3$

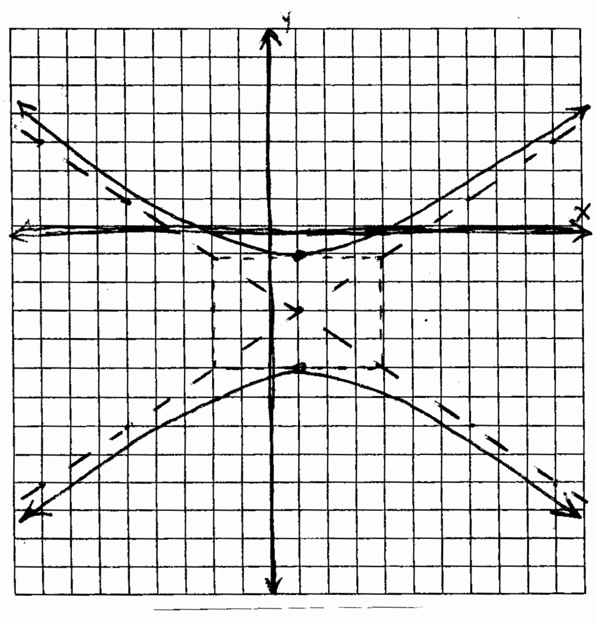
center $(1, -3)$

x-direction + y-direction: denominators

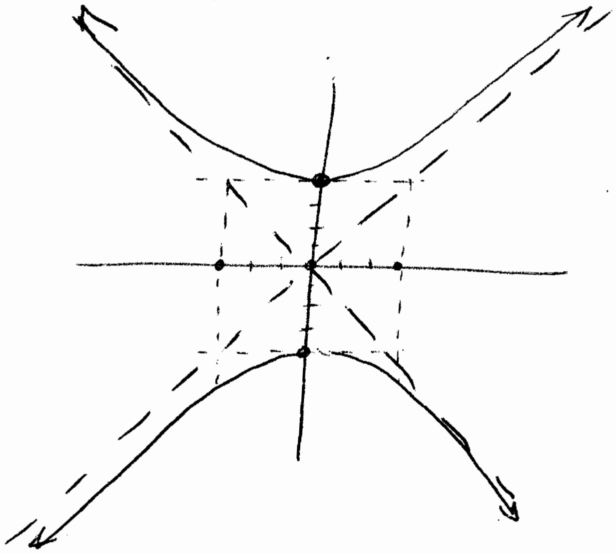
x-direction: under x^2

take square root $\sqrt{9} = 3$

y-direction: under y^2 $\sqrt{4} = 2$



SKIP (6) Sketch $\frac{y^2}{16} - \frac{x^2}{9} = 1$

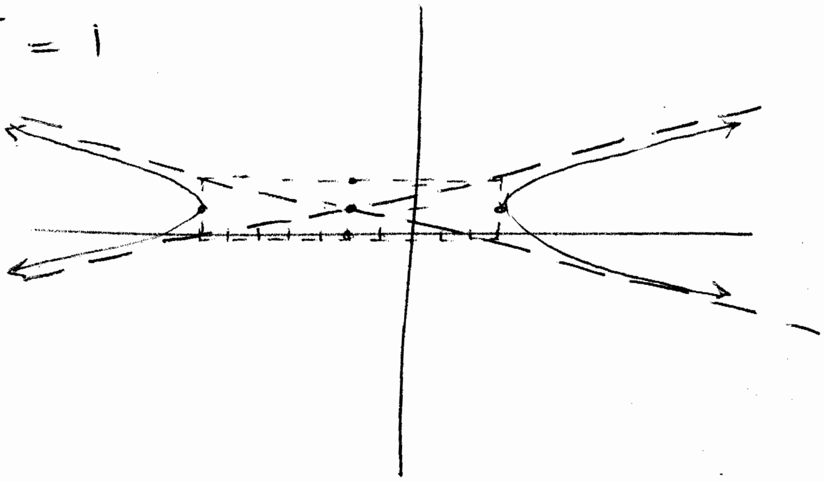


center $(0,0)$
x dir $\sqrt{9} = 3$ (under x^2)
y dir $\sqrt{16} = 4$ (under y^2)

$y^2 - x^2$ up/down

CAUTION:
Branches of hyperbola do not cross asymptotes and do not go inside the box.

(7) Sketch $\frac{(x+2)^2}{25} - (y-1)^2 = 1$



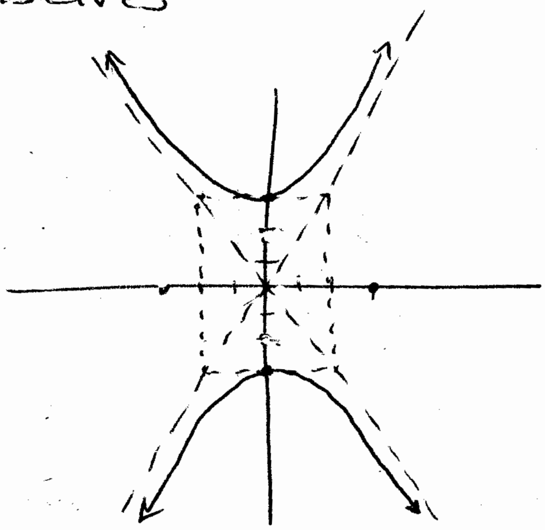
$x^2 - y^2$ left/right
center $(-2,1)$
x-dir $\sqrt{25} = 5$
y-dir $\sqrt{1} = 1$
$x^2 - y^2$ left/right

(8) Sketch $4y^2 - 9x^2 = 36$

Notice: not 1 on RHS. Focus here and all other coefficients will fix themselves

$$\frac{4y^2}{36} - \frac{9x^2}{36} = \frac{36}{36}$$

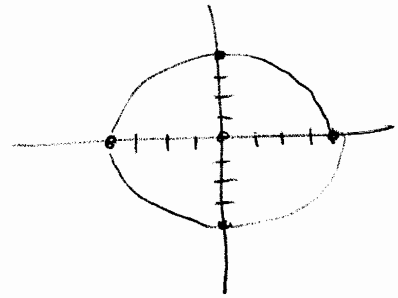
$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$



$y^2 - x^2$ opens up/down
Center $(0,0)$
x-direction 2, y direction 3

⑨ Sketch $x^2 + y^2 = 16$

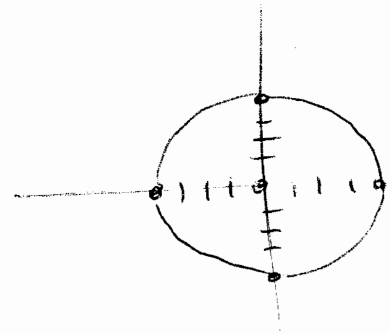
Method 1. Notice it's a circle
center $(0,0)$
radius $\sqrt{16} = 4$



Method 2. Graph as you would an ellipse: divide to get 1 on RHS

$$\frac{x^2}{16} + \frac{y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{16} + \frac{y^2}{16} = 1$$



center $(0,0)$
x direction $\sqrt{16} = 4$
y direction $\sqrt{16} = 4$ also

⑩ sketch $x = -y^2 + 6y$

x and y^2 $\Rightarrow$ parabola left/right

$a = -1$ $\Rightarrow$ opens left (neg. x direction)

vertex $y = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$

$x = -(3)^2 + 6(3) = 9$

check

$x = -(y-3)^2 + 9$

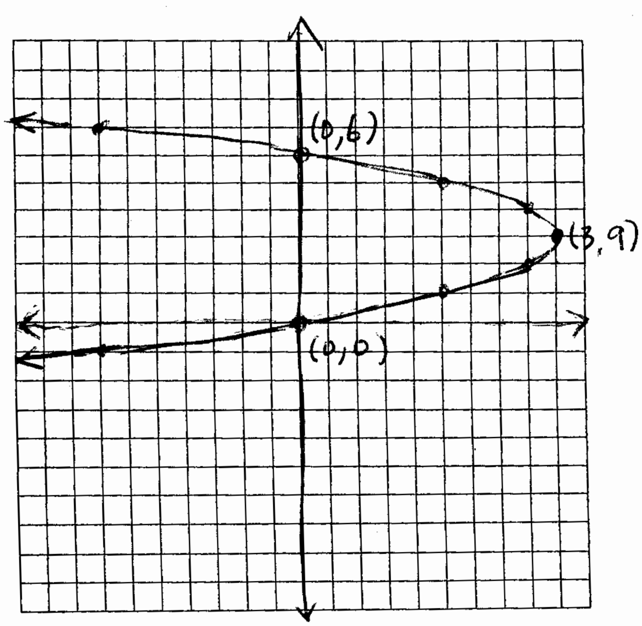
$= -(y^2 - 6y - \dots$

$= -y + \dots$

$= -y^2 + 6 \dots$ ✓

CAUTION vertex $(9,3)$

not $(3,9)$!



y-ints? set $x=0$

$0 = -y^2 + 6y$

$y^2 - 6y = 0$

$y(y-6) = 0$

$y = 0, 6$

Sketch graph.

(11) $\frac{(x-7)^2}{25} + \frac{y^2}{81} = 1$

added ellipse

$= 1$ ☺

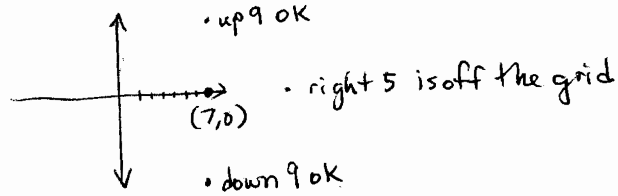
center $x = 7$ $y = 0$ $\Rightarrow$ $(7, 0)$

$(x-7)^2$ $(y-0)^2$

x-direction $\sqrt{25} \Rightarrow 5$ left and right from center

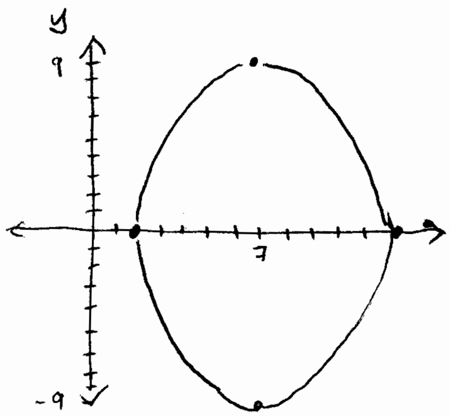
y-direction $\sqrt{81} \Rightarrow 9$ up and down from center

Numbers are rather large ... let's imagine our axes and grid



Need more XMAX, less XMIN

Move y-axis to left.



Sketch graph.

(12) $4y^2 + 49x^2 = 196$

no fraction

add ellipse

no fraction

not = 1

$\circledast$ FOCUS ON MAKING RHS = 1

∴ divide all terms by 196

$$\frac{4y^2}{196} + \frac{49x^2}{196} = \frac{196}{196}$$

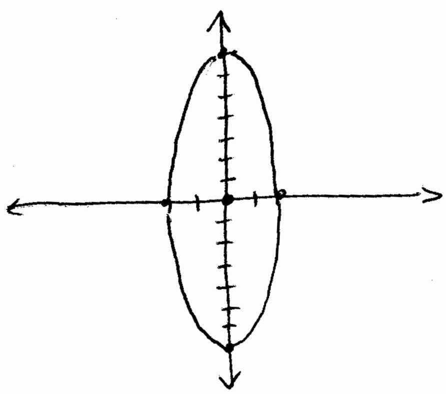
use $\boxed{\text{MATH}}$ to reduce fractions
>frac

$$\frac{y^2}{49} + \frac{x^2}{4} = 1$$

write x^2 first

$$\frac{x^2}{4} + \frac{y^2}{49} = 1$$

ellipse, center $(0,0)$, x-direction $\sqrt{4} = 2$
y-direction $\sqrt{49} = 7$



Sketch graph.

(13) $49x^2 - y^2 = 49$

subtracted hyperbola

$\neq 1$

☹ * FOCUS ON MAKING RHS = 1

divide all terms by 49

$$\frac{49x^2}{49} - \frac{y^2}{49} = \frac{49}{49}$$

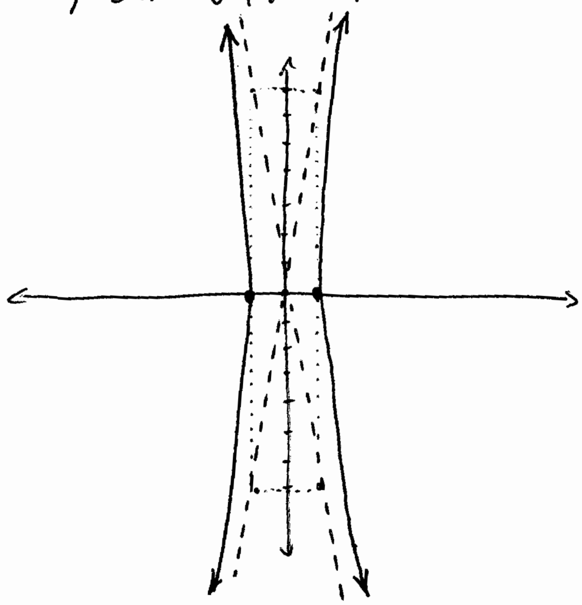
$$\frac{x^2}{1} - \frac{y^2}{49} = 1$$

x's first ⇒ opens left/right

Center (0,0)

X-dir $\sqrt{1} = 1$

Y-dir $\sqrt{49} = 7$



10.2.47 **The graph of the given equation is an ellipse. Find the distance between the x-intercepts and the distance between the y-intercepts. Decide which distance is longer. How much longer is the longer distance than the shorter distance?**

$$\frac{x^2}{4}+\frac{y^2}{49}=1$$

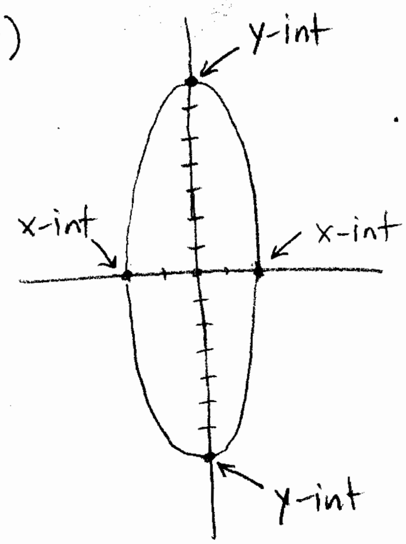
Determine which distance is longer.

- ○ **distance between the x-intercepts**
- ✔ **distance between the y-intercepts**

How much longer is the longer distance than the shorter distance?

10 units

center (0,0)
x-dir 2
y dir 7
graph



distance between x-ints:
from −2 to +2
= 4

distance between y-ints:
from −7 to +7
= 14

subtract
$14-4=\boxed{10}$

10.2.61 A planet's orbit about the a certain star can be described as an ellipse. Consider this star to be the origin of a rectangular coordinate system. Suppose that the x-intercepts of the elliptical path of the planet are ± 150,000,000 and that the y-intercepts are ± 135,000,000. Write the equation of the elliptical path of the planet.

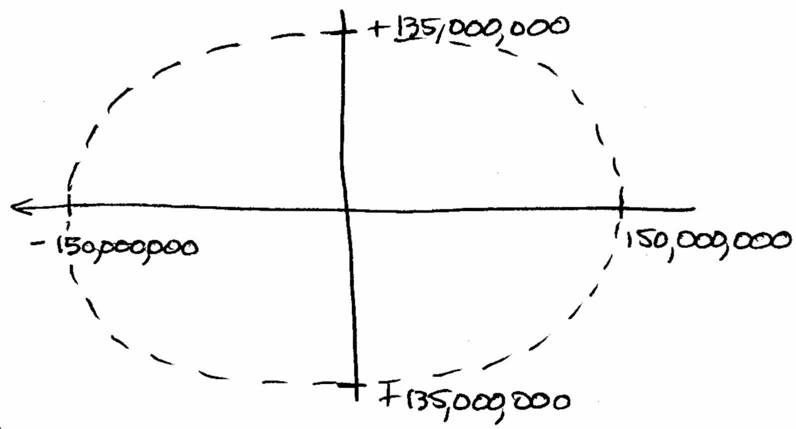
The eliptical path of the planet is $\frac{x^2}{\square} + \frac{y^2}{\square} = 1$.

Sketch the given information:

x-intercepts: + ~~1~~150,000,000 and −150,000,000

y-intercepts: + 135,000,000 and −135,000,000.

"elliptical" means "ellipse"



The center must be (0,0).

So we're trying to find the denominators

$$\frac{x^2}{\square} + \frac{y^2}{\square} = 1$$

We take the square root of this number to get the distance from the center to the ellipse. That distance is

ditto

square 135,000,000 to get

18,225,000,000,000,000

$\sqrt{\square} = 150{,}000{,}000. = 1.5 \times 10^8$

Square both sides! $(1.5 \times 10^8)^2 = 2.25 \times 10^{16}$

22,500,000,000,000,000